Teachers as Designers of Effective Numeracy Tasks

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In this paper, preliminary research into an effective approach to numeracy task design is described and analysed using a conceptual framework based on two dimensions – one related to the nature of numeracy and the other concerned with the characteristics of effective mathematical tasks. A case that documents one teacher's attempt to create a numeracy task for her class of preparatory students is used to illustrate the relevance of the components of the framework to task design from the perspectives of participant teachers. Teachers' comments provided insight into aspects of the design process that require greater emphasis and also helped identify complexities that require resolution before a coherent framework for effective numeracy task design can be realised.

Introduction and Background

Numeracy, or mathematical literacy as it is sometimes known internationally, is the capacity to make effective use of mathematics in personal life, the workplace, and when exercising civic responsibilities. What is means to be numerate is in a state of constant revision because of the mathematical demands of an increasingly globalised world characterised by rapid technological and economic change (Academy of Sciences, 2016; Office of the Chief Scientist, 2012). The challenges associated with maintaining a numerate citizenry have been recognised in government reports and international assessment frameworks of numeracy. For example, the Programme for the International Assessment of Adult Competencies (PIACC) recently identified numeracy and problem solving in technology-rich environments as necessary for participating in the labour market, education and training, and social and civic life (Organisation for Economic Co-operation and Development, 2013).

Numeracy has been a national education priority for more than a decade (e.g. Ministerial Council on Education, Employment, Training and Youth Affairs, 1989) and is a general capability within the Australian Curriculum, Assessment and Reporting Authority (2016), yet, at best, there appears to be little progress in students' numeracy performance in recent years. Results from the Programme for International Student Assessment (PISA), for example, indicate that too many Australian students fail to meet numeracy benchmarks. These results show that 20% of Australian 15 year olds did not meet the international proficiency Level 2 for mathematical literacy – indicative of the level of competence necessary to use mathematics effectively in real-life situations. Further, 43% of students were placed below the Australian nationally agreed baseline of Level 3 (Thomson, De Bortoli, & Buckley, 2013). When compared to other countries, Australia was ranked 19th for mathematical literacy in 2012, down from 13th in 2009 and 8th in 2006. If the root cause of these results remains unattended, Australia faces the prospect of limited life opportunities for individuals and the diminished effectiveness of our work force, resulting in a potential down-turn in our nation's growth and prosperity.

Burkhart and Swan (2013) argue for the importance of task design in improving the teaching of mathematics because tasks are integral to many dimensions of mathematics

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learning, including mathematical content, processes and modes of working. Evidence that coherent research and development approaches to task design are effective in improving teaching practice is provided by the long-term success of programs such as Connected Mathematics (Lappan & Phillips 2009). At the same time, Schoenfeld (2009) opinions that the principles of task design are rarely made explicit, and so it is difficult to for others, including teachers, to adopt effective approaches to task creation and adaptation. This is especially so in the case of numeracy where limited research exists on the nature of quality numeracy tasks and how these are designed. Thus, the improvement of students' numeracy capabilities requires improved knowledge of how numeracy tasks are designed and implemented in school classrooms in all subjects, not just mathematics (Steen, 2001).

The purpose of this paper is to report on aspects of preliminary findings from a three-year longitudinal study that aims to develop and trial principles of numeracy task design and investigate the extent to which these principles can assist teachers in developing effective numeracy tasks. In doing so the following research question will be addressed:

What principles of design do teachers employ when creating or adapting tasks for promoting numeracy across the curriculum?

Conceptual Framework

The conceptual framework that underpins this study is based on two complementary dimensions (1) the nature of numeracy, and (2) the characteristics of effective mathematical task design. The nature of numeracy is captured in the *Model of Numeracy for the 21st Century* (Goos, Geiger & Dole, 2014) with the characteristics of effective mathematical task design synthesised from relevant literature as *generic principles of task design*. The components of each of these dimensions were used as a starting point for discussion of effective numeracy task design and for the development of trial tasks with project teachers.

The Model of Numeracy for the 21st Century has been validated through a series of research projects (e.g. Goos, Dole, & Geiger, 2014; Geiger, Forgasz & Goos, 2015; see Table 1).

Table 1
The Model of Numeracy for the 21st Century

Mathematical knowledge	Mathematical concepts and skills; problem solving strategies; estimation capacities.
Contexts	Capacity to use mathematical knowledge in a range of contexts, both within schools and beyond school settings
Dispositions	Confidence and willingness to use mathematical approaches to engage with life-related tasks; preparedness to make flexible and adaptive use of mathematical knowledge.
Tools	Use of material (models, measuring instruments), representational (symbol systems, graphs, maps, diagrams, drawings, tables, ready reckoners) and digital (computers, software, calculators, internet) tools to mediate and shape thinking
Critical orientation	Use of mathematical information to: make decisions and judgments; add support to arguments; challenge an argument or position.

This model brings together vital aspects of numeracy identified in previous research (e.g. Hogan, Van Wyke & Murcia, 2004) while privileging the role of critical mathematical thinking within personal and collective functional and socio-politically framed scenarios. The model incorporates four components, contexts, mathematical knowledge, tools, and dispositions, through which are intertwined an analytical and evaluative capability – a critical orientation. Aspects of the model are summarized here but described more fully in other publications (e.g. Geiger, Goos & Dole, 2015).As there is limited literature available specific to the characteristics associated with the design of effective numeracy tasks, generic principles were identified via a synthesis of relevant research literature. These characteristics are outlined and described in Table 2. Further elaboration upon the development of these principles is available within previous publications (e.g. Geiger, Goos, Forgasz & Bennison, 2014).

Table 2
A Synthesis of Characteristics Associated with the Design of Effective Numeracy Tasks

Fit to circumstance	Accommodating curriculum requirements and other affordances or constraints within a school setting, for example, teaching materials available within a particular school.
Challenge	Extending students' thinking by including elements of challenge in tasks provides opportunity for reasoning, risk taking, and the justification decisions.
Challenging yet accessible	Tasks must feel achievable to all students regardless of their prior history of achievement.
Complementary pedagogies	The pedagogical approach must match the demands and instructional intention of the task.
Transparent	In order for students to engage fully with tasks, activities must not only be accessible but also transparent in relation to expected outcomes – there is clarity around what is required of students to achieve success.
Opportunity to make decisions and judgments	The opportunity to make decisions and judgments introduces a critical demand into a task and provides purpose for students to engage with an activity.
Iterative cycles of design and improvement	To ensure the quality of a task, activities must also be developed, appraised, trialled, evaluated, and retrialed. This means effective activities will take time to develop and require a commitment to reflective practice.

Research Design

This project employs a design-based research approach, as the aim is to develop and test theory about how teaching practice and student learning change in response to a teacher professional learning program involving iterative interventions (Cobb, Confrey, DiSessa, Lehrer, & Schauble, 2003). Design-based approaches are particularly suitable for studies that include interventions in classrooms that are known to be complex, interactive, and reflexive in nature (Schoenfeld, 2006). The teacher professional learning component of the project is underpinned by the Loucks-Horsley, Love, Stiles, Mundry, and Hewson's (2003) framework

for professional development. This framework situates effective professional learning within school-based contexts where teachers can try out and validate ideas in their own classrooms, a complementary approach to design-based research. The research design comprises of overlapping three-phase interventions documented via case studies of teachers and students, used to identify changes over time.

Whilst Phase 1 has an interpretive focus in the sense that the goal is to gain insight into and understanding of the practices of exemplary teachers as they design and implement numeracy tasks in classrooms within specific subject contexts, Phase 2 is interventionist and iterative, attempting to influence the development of a group of teachers new to the project. Phase 3 seeks to establish the benefit to students' numeracy learning of approaches based on subject specific numeracy tasks and associated pedagogies. This paper is based on preliminary data drawn from the first part of Phase 1 of the project which involved two whole project teacher meetings and a round of school visits that included classroom observations, student interviews and a stimulated recall interview with teachers.

Participants

Five teachers, along with intact class groups, were recruited for the initial phase of the study from both Queensland and Victorian schools. Teachers were purposively selected (Burns, 2000) from a pool the researcher had worked with in previous research projects. Invitations to participate in the project were based on participating teachers' effectiveness as designers of numeracy tasks (established in earlier projects) in a range of learning areas and sectors of schooling – Secondary English, Mathematics, Technology and Design, and Early Childhood/Primary teaching.

Cycle of Events

In the first workshop, the Numeracy for the 21st Century (Goos, Dole & Geiger, 2014) was described and generic principles of task design outlined. Immersion experiences based on tasks developed through previous numeracy projects (e.g. Geiger, Goos & Dole, 2015), where activities are situated within subjects other than mathematics, were presented and deconstructed by the researcher, in collaboration with teachers, against the Model of Numeracy for the 21st Century and the generic principles of mathematics task design. The remainder of the workshop was devoted to establishing goals, initial development of classroom numeracy tasks, and planning for task trials.

Between the first and second workshops, teachers implemented trial tasks in their own classrooms. Lessons where tasks were implemented were observed and video-recorded during school visits by the researcher. Teachers and focus groups of students were interviewed after each observation to gain their perspectives on the level of student interest and engagement with tasks as well as their perceived effectiveness. Teacher interviews were in the form of individual video stimulated recall sessions (e.g. Muir & Beswick, 2007) where recordings were reviewed, with the researcher, for the purpose of providing insight into the creative or adaptive processes teachers' employed generate tasks and select pedagogies.

A second whole project meeting took place after the first cycle of task design and implementation, lesson observations and video stimulated recall sessions. This meeting provided: opportunity for teachers to report upon and discuss their experience of designing and implementing tasks; an avenue for the researcher to report on insights gained through video stimulated recall sessions; opportunity for conversation about other teachers'

implemented activities; time for discussion of principles of numeracy task design (PNTD); opportunity to plan for another round of trialling and implementation of new tasks based on PNTD.

Insights gained from the first round of school visits and the second whole project meeting are presented in the following sections.

A Reflective Discussion on Task Design in relation to the Numeracy Model

Olive was an early childhood teacher in a Catholic primary school situated in a regional location. She had developed a series of tasks for her group of preparatory students (typically 5-6 years of age) as part of an activity rotation. All tasks were set in a context related to building a prayer garden on recently acquired land adjacent to the school. In one of these tasks, students were asked to determine if a long rectangular bench seat could be moved to a different position, without moving the seat itself. No formal measuring tools were available (e.g. tape measures, rulers…) but 30 cm by 30 cm square tiles were provided among other equipment the teacher had set aside for the task.

After the teacher had explained the task, students discussed among themselves how they could go about the activity. After this discussion, students used the tiles to determine a measure of the bench seat length by placing the tiles end-to-end across the top of the seat. Once the length of the bench seat had been covered, the tiles were gathered up and moved by the students to the proposed new site for the seat. Once in position, at one end of the designated space, students laid out the tiles end-to-end on the ground until they reached the other end of the assigned area. There were left over tiles, which were then piled up on top of the last laid out on the ground. When Olive asked students to for a response to the question posed in the task, they concluded that there was not enough space to move the bench seat to the proposed space and that another place would need to be found.

After observing the lesson, the researcher reviewed the video recording of the activity with Olive. During the review, the researcher asked Olive about how she came to think of the task in her lesson. According to Olive, the design process began by reflecting firstly on the content of relevant curriculum documents she needed to address at that time of the year. At the same time, she also considered what resources or aspects of the environment could be utilised. Olive believed it was very important to know relevant curriculum documents well across all subject areas (in her situation as an early childhood teacher). This allowed her to pick out different strands from both geography and religious education to design a task that dovetailed with an opportunity made available by the purchase of a new property and the school plans to build a prayer garden.

In terms of the Model of Numeracy for the 21st Century, Olive had selected a curriculum *context* that involved aspects of both geography and religious education. She had also taken advantage of a *context* related to the changed physical environment related to her school – the purchase of land for future school expansion and the plan to use part of the land as a prayer garden. The activity required the use of *mathematical knowledge* in that the concept of length was vital to the task as was the use of informal measures. Olive was attempting to promote students *dispositions* by providing a task that connected with students' interests and required of them the flexible use of *mathematical knowledge* – knowledge they had been exposed to in a classroom setting but now in an unfamiliar *context*. Students made use of square tiles as measuring *tools*, being too young to use formal means of measurement. The task called for students to make a judgement about the possible relocation of the bench seat and so required them to take a *critical orientation* to the proposed scenario.

When asked if she had use the Model of Numeracy for the 21st Century when developing the task, Olive stated it had been used as a checklist for aspects to include in tandem with her development of the task. While she had begun her design process by thinking about mathematics related aspects of the school's geography and religious education programs, additional elements were developed into the task via a reflexive process between the activity and the Model of Numeracy for the 21st Century.

Olive was also asked if she had utilised the generic principles of task design (also discussed in the first workshop) but indicated that these had been somewhat of an after thought. Discussion with Olive about these principles in relation to the task, however, revealed that they had been addressed, even if coincidentally.

Collaborative Reflection on Aspects of Task Design

At the second whole project meeting, teachers agreed to share video excerpts from their lessons in order to offer and receive critique of tasks implemented during the researcher school visit. The purpose of this activity was to determine if components of the two dimensions of the conceptual framework could be identified by "others" – teachers who were not responsible for developing a task. By coincidence, Olive was not able to attend this session but gave consent for the presentation of a video except from her lesson with commentary by the researcher. After playing the video excerpt, the researcher asked that comments be related to the design of the task, firstly by referring to the two dimensions employed as starting points (i.e. the Model of Numeracy for the 21st Century and the generic principles of task design), and secondly any new aspects or dimensions not covered by these organising constructs. Teachers were able to quickly pick aspects of the task that aligned with the Model of Numeracy for the 21st Century in a similar way to Bec herself. They were also able to identify elements of the task that were consistent with the generic principles of task design.

The first aspect noticed was that of *challenge*:

> Doug: It definitely engages students and definitely challenged them…She didn't give many answers, so she said, "How do you think you might propose to go about the task?"
>
> Susan: Taking them outside set another challenge as well, because as soon as you take small children outside there's lots of things to look at apart from the task. And some kids need that on paper, looking at it and manipulating, but other kids need that outside physically moving. So that's another aspect of challenge.

These comments clearly indicate the other members of the group recognised the task as *challenging* but also identified aspects that were not associated with mathematical *challenge*, for example, managing small children in an environment that was an unfamiliar learning space.

The group could also see how the task fit the circumstance both from the perspective of curriculum and the physical environment.

> Doug: … it really fit to the circumstance of the context of the school… now they had some real life context.

While Doug recognised the *fit to circumstance* aspect of the task he has, interestingly, linked this to *context* – a dimension from the Model of Numeracy for the 21st Century.

Decision-making was also seen to be integral to the task:

> Susan: They had plenty of opportunity to make decisions pretty much about anything that they wanted to do within their little task card, so that was really good…And she gave them a lot of freedom because when they were looking at the seats, they would've been looking at all the dimensions. The more traditional way would be to lead them to have a look at this [particular] dimension, whereas she stepped back from that.
>
> So that was incredibly important because they're the steps that then people can't do on their own because someone else has done it for them… how often do teachers say so much that all the challenge is gone, and by not saying it then she learnt they didn't understand that so then I can take it another step rather than just going the whole way and saying, "Here's how you do it".

In this comment, Susan makes a link between the *opportunity to make decisions and judgments* and the *pedagogy* required to *complement* such activity – in this case an investigative approach. Susan's comments also are indicative of the tension between *challenge* and *accessibility* – related to how making a task *transparent* can impact on the *challenge* of an activity and so how and when teachers should or shouldn't intervene.

Teachers' comments indicate that all generic principles of design were evident to the group with the exception of *iterative cycles of design and improvement*, which was not relevant in the case of an initial trial of an activity.

Conclusion

While there exists convincing argument for the important role of task design in improving students' mastery of mathematical content, thinking processes and modes of working (Burkhart & Swan, 2013), the principles of design that underpin effective tasks have rarely been made transparent (Schoenfeld, 2009). Without the foundations of how tasks can be created or adapted it is difficult for teachers to develop effective approaches to developing activities for their own classrooms – particularly in the case of numeracy tasks which demand a broader range of skills than is required for the production of traditional mathematical tasks.

In this paper, preliminary research into an effective approach to numeracy task design was described and analysed using a conceptual framework based on two dimensions – one related to the nature of numeracy and the other concerned with the characteristics of effective mathematical tasks. A case that documents one teacher's attempt to create a numeracy task for her class of preparatory students was used to illustrate the relevance of the components of the framework to task design from the perspectives of participant teachers. Teachers' comments provided insight into aspects of the design process that require greater emphasis and also helped identify complexities that require resolution before a coherent framework for effective numeracy task design can be realised.

A comprehensive knowledge of relevant curriculum documents was considered to be vital by the teacher who created the bench seat task. This allowed her to think in a genuinely cross-curricular fashion when connecting curriculum requirements to available resources and opportunities – a challenging demand for primary and early childhood teachers who teach across multiple subject areas. How this demand relates to the circumstances of secondary teachers, who may be responsible for only one or two subjects, when they attempt to create numeracy tasks, needs further investigation.

While teachers, during the whole project meeting, were able to associate dimensions of each framework with the task, sometimes the distinction between these elements was unclear. For example, there appears to be a close intertwining of context and fit to circumstance across the two frameworks and accessibility and challenge within the generic

principles of design. This suggests that work is required on the dimension descriptor that makes these elements more distinct, or that some dimensions should be combined within a single more coherent framework.

These two issues have emerged from the very early stages of this project and point to immediate matters that require resolution before achieving the goal of a coherent set of principles for numeracy task design. Further research will doubtless reveal additional issues, especially when these principles are trialled with new teachers who will be introduced into the group during the next phase of the project.

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